PVP 12

Code: EC3T2

II B.Tech - I Semester – Regular Examinations - December 2014

PROBABILITY THEORY AND STOCHASTIC PROCESS
(ELECTRONICS & COMMUNICATION ENGINEERING)

Duration: 3 hours Marks: 5x14=70

Answer any FIVE questions. All questions carry equal marks

1. Let X be a discrete r.v whose cumulative distribution function is $F(x)=\begin{cases}0 & for\ x<-3\\ 1/6 & for\ -3\le x<6\\ 1/2 & for\ 6\le x<10\\ 1 & for\ x\ge 10\end{cases}$

   a) Find $P(X\le 4)$, $P(-5<X\le 4)$, $P(X=-3)$, $P(X=4)$. 7 M

   b) Find the probability mass function. 7 M

2. a) If a poisson variate X is such that P(X = 1) = 2P(X = 2). Find P(X = 0) and var (X). If X is a uniform random variable in [-2, 2], find the p.d.f of $Y=|x|$ and E[Y]. 7 M

   b) Determine the real constant a, for arbitrary real constants m and 0<b, such that $F_x(x)=ae^{-|x-m|/b}$ is a valid density function. 7 M

3. a) Describe negative binomial distribution X~NB (k, p) where X=number of failures preceding the k^{th} success in a sequence of Bernoulli trials and p=probability of success. Obtain the MGF of X, mean and variance of X. 7 M

b) Find the moment generating function of an exponential random variable and hence find its mean and variance. 7 M

4. a) If X and Y are random variables having the joint density function

$$f(x,y) = \frac{1}{8}(6 - x - y);\ 0 < x < 2;\ 2 < y < 4$$
$$= 0,\ otherwise$$
7 M

Find i) $P(X < 1 \cap Y < 3)$

ii) $P(X + Y < 3)$

iii) $P(X < 1 / Y < 3)$.

b) A random sample of size 100 is taken from a population whose mean is 60 and the variance is 400. Using Central limit theorem, with what probability can we assert that the mean of the sample will not differ from $\mu = 60$ by more than 4? 7 M

5. a) Find the mean value of the function $g(X,Y)=X^2+Y^2$ where X and Y are random variables defined by the density function $f_{X,Y}(x,y)= e^{-(X2+Y2)/2\sigma^2}/2\Pi\sigma^2$. 7 M

b) For two zero-mean Gaussian random variables X and Y, show that their joint characteristic function is $\phi_{x,y}(w_1,w_2)=\exp\{-1/2[\sigma_x^2 w_1^2+2\rho\sigma_x\sigma_y w_1 w_2+ \sigma_y^2 w_2^2]\}$. 7 M

6. a) Given that WSS random process $X(t)=10\cos(100t+\theta)$, where θ is uniformly distributed over $(-\pi,\pi)$ is correlation ergodic. 7 M

b) If {X(t)} is Gaussian process with $\mu(t)=10 \text{ and } C(t_1,t_2)=16e^{-|t_1-t_2|}$, find the probability that i) $X(10)\leq 8$ ii) $|X(10)-X(6)|\leq 4$. 7 M

7. a) Given that a process X(t) has the auto correlation function $R_{XX}(\tau)=Ae^{-\alpha|\tau|}\cos(\omega_0\tau)$ *where* $A>0, \alpha>0$ *and* ω_0 are real constants, find the power spectrum of X(t). 7 M

b) If the cross-correlation of two processes {X(t)} and {Y(t)} is $R_{XY}(t,t+\tau)=\frac{AB}{2}[\sin(\omega_0\tau)+\cos(\omega_0(2t+\tau))]$ where A,B and ω_0 are constants. Find the cross power spectrum. 7 M

8. a) Calculate the power spectral density of a stationary random process whose auto correlation is $R_{XX}(\tau) = e^{-\alpha|\tau|}$ 7 M

b) Two identical networks are cascaded. Each has impulse response h(t)=u(t)3t exp(-4t). A wide sense stationary process X(t) is applied to the cascade's input. 7 M

i) Find an expression for the response Y(t) of the cascade.

ii) If E[X(t)]= 6, find E[Y(t)].